CHAPTER 36 Mirrors and Lenses and Their Uses

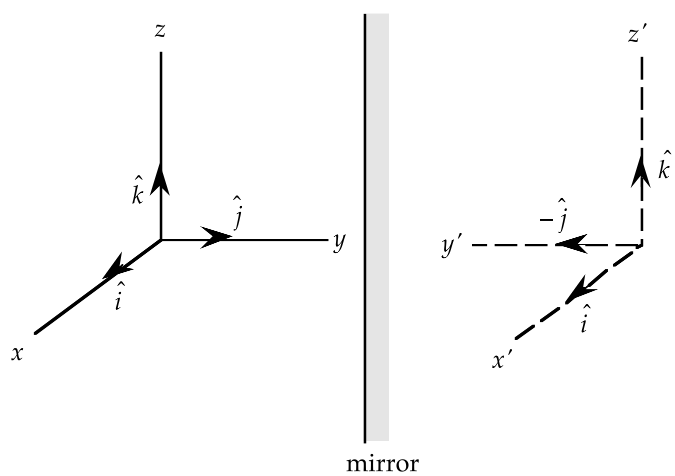
Answers to Understanding the Concepts Questions

1. This sign is meant for drivers who see a vehicle with flashing lights arrive from behind and read the message "AMBULANCE" in their rear-view mirrors. Try it!

2. The mirrors reflect, rather than absorb, most of the light. If you are inside the mirrored room and you put your hand up, for example, your hand will be illuminated not only with the direct light from the candles but also with the reflected candlelight from the mirrors. So it appears more brightly illuminated. The mirrored room does indeed appear brighter.

3. *Any* portion of the lens will give the same image, because all that counts is the curvature of the surfaces, not how big those surfaces are. We have found it convenient to draw principle rays that require the use of a "complete" lens, but remember that any ray will end up where the principle rays do in an ideal lens.

4. In the diagram below, a right-handed xyz coordinate system becomes left-handed upon mirror reflection, as the unit vectors in the x- and z-directions are unchanged while that in the y-direction is reversed. Here we assumed the special case when the y-axis is perpendicular to the mirror surface, but it is not hard to verify that the reversal from right/left-handedness to left/right-handedness is a general feature of any mirror reflection.



5. From the plot above for Question 4 we see clearly that the mirror reflection leaves unchanged a coordinate whose axis is parallel to the mirror surface, while it reverses the coordinate whose axis is perpendicular to the mirror surface. In the present case both x- and y- axes are parallel to the mirror surface so x and y remain the same upon mirror reflection, while z becomes $-z$. The correct rule is therefore $(x, y, z) \rightarrow (x, y, -z)$. The physicist would indeed be surprised.

6. The dentist will typically want an enlarged view of your tooth in circumstances where the mirror is close to the tooth. Figure 36-10 shows that what the dentist will use for this purpose is a concave

mirror. As the discussion on convex mirror (pages 1004 – 1006) indicates, a convex mirror always gives an image reduced in size from the object.

7. It is a convex mirror, which can only produce an upright image that's smaller than the object — and hence the warning label. The advantage over a plane mirror is a wider field of view; by shrinking the sizes of each image we can see more of what's behind us.

8. All these lenses obey Eq. (36-2), which states that when an object is placed at the focal point ($s = f$), the image is at infinity ($i = \infty$). Eq. (36-6) then gives the magnification to be infinite. Ray-tracing will give the same result: The image is infinitely far away and infinitely large. The crucial feature that makes this true is that rays from an edge of the ball that lies above the focal point will trace out a path that is not quite parallel to the optical axis. At infinity, where the rays from the off-axis part of the object arrive at an image, they are infinitely far above the optical axis. Of course, if there were an image of finite size infinitely far away, its angular size would be zero and you could not see it!

9. The spherical surface is essentially a convex mirror in which images are smaller than the objects, allowing a wider field of view to fit into the surface area of the mirror.

10. The image formed by a convex mirror is always smaller than the object. When the object is infinitely far from the mirror, its image size is the smallest, as is common sense. As we bring the object closer to the mirror, the size of the image grows, approaching that of the object itself as it is brought right up against the mirror. So to make the image taller the object in Fig. 36-15 should be moved to the right, so as to get closer to the mirror. The correct answer is (b).

11. The amount of refraction depends on the indices of refraction of both media involved. Although we derived the lens-maker's formula assuming one of these media had an index of refraction equal to one, we could rederive the formula assuming the exterior medium to be water, $n = 1.33$. We would then find a different result for the focal length, the distance at which rays from infinity are brought to a focus.

12. The magnifying glass is a convex (converging) lens. Since the light rays from the Sun are essentially parallel, they converge at the focal point of the lens, a distance f behind it. Here f is the focal length of the lens. Since all the sunlight intercepted by the lens converges onto a much smaller dot, the energy of the sunlight is much more concentrated on the spot than elsewhere, and the heat it produces makes the spot hot.

13. The camera couldn't care less about whether the object being photographed is virtual or real. It uses rays coming into the lens, and whether those rays actually come from a real object or only are aligned as if they are coming from a virtual object do not come into play in the formation of the final image on the film plane.

14. Yes. Glasses designed to correct near-sightedness are made of concave (diverging) lens; these do not focus a beam of parallel sunlight to a concentrated spot (like a convex lens would) to raise the temperature sufficiently to produce fire. Rather, they diverge the beam so that it appears to come from a point in front of the lens; this is the virtual focal point.

15. Not very. A ray coming in along the axis continues along the axis, and generally the principal rays all overlap. The one exception is the case that the originating point is the focus; here we can use the principal ray that comes from the focus and goes out parallel to the axis.

16. Since their eyesight has not improved, the focal length of the thin glasses must be identical to that of the thick ones. The focal length, in turn, is determined by the radii of curvature of both the front and back surfaces, and n, the index of refraction of the material of which the glasses are made. With thicker glasses it is easier to curve the surfaces more to shorten the focal length, i.e., to make the lenses

more powerful for people with severe cases of near- or far-sightedness. With thinner glasses, to maintain the same (short) focal length, a material with a greater value of n is the solution.

17. When the pupil is dilated, light rays can strike the outer edge of the lens, not just the central portion. This accentuates any imperfection of the lens, making it easier for the examiner to identify certain vision defects.

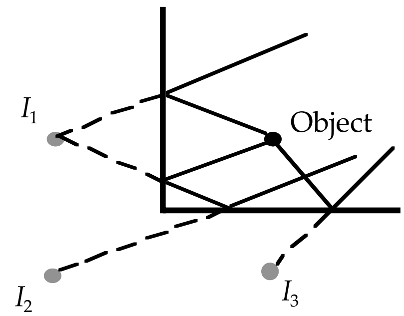
18. Let's call the direction out from the eye the z-direction. We would start by tailoring the glass so that there is a central point O in the xy-plane and, unless astigmatism is involved, we do what ever is done as a function only of the distance out from the central point, that is, radially. Is there any tailoring to be done in the z-direction? Yes; if not, any ray would be passing though a flat plane of glass of fixed index of refraction and would be displaced but not changed in direction. If, say, it is necessary for eyesight correction to make rays from infinity converge closer to the lens, then we would want the index of refraction of the glass to decrease as we move through the glass toward the eye. This would bend the rays towards point O, which is the effect desired.

19. The legend is rather doubtful. To focus the reflected sunlight at the enemy ships the ships must be one focal length away from the mirror, whose radius of curvature must then be exactly twice the distance to the ships. This would work only when the enemy ships are at a specific distance from a given mirror. And even if this were the case, any movement of the ships that changes their distances to the mirror would spoil the effect.

20. Yes. The image is smaller than the object as long as the object is placed at a distance more than twice the focal length away from the lens. As an extreme example, just consider the image of the Sun formed at the focal point of a magnifying glass (a convex lens).

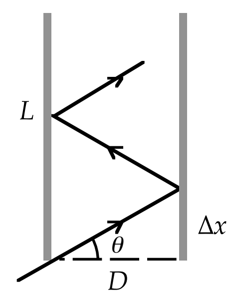
Solutions to Problems

1. The image formed by light that reflects from one mirror forms a virtual image a distance behind the mirror equal to the distance of the object in front of the mirror. Each mirror will produce such an image. Light that reflects from both mirrors will produce a third image, as seen in the diagram. This image can be thought of as the image produced by either of the other images by reflection from the (extended) mirror. Note that only a few rays have been drawn, since we know that the image is as far behind the mirror as the object is in front of the mirror.
   There are $\boxed{\text{3 images}}$.



2. The source is a distance $s_1 = a - x_0$ from the mirror at $x = a$, so the image is behind the mirror a distance
   $i_1 = s_1 = a - x_0$. Its location is $x_1 = a + i_1 = a + a - x_0 = \boxed{2a - x_0}$.
   The source is a distance $s_2 = a + x_0$ from the mirror at $x = -a$, so the image is behind the mirror a distance
   $i_2 = s_2 = a + x_0$. Its location is $x_2 = -a - i_2 = -a - a - x_0 = \boxed{-2a - x_0}$.
   The first image is a distance $s_3 = 2a + i_1 = 3a - x_0$ from the mirror at $x = -a$, so the image is behind the mirror a distance
   $i_3 = s_3 = 3a - x_0$. Its location is $x_3 = -a - i_3 = -a - 3a + x_0 = \boxed{-4a + x_0}$.
   The second image is a distance $s_4 = 2a + i_2 = 3a + x_0$ from the mirror at $x = a$, so the image is behind the mirror a distance
   $i_4 = s_4 = 3a + x_0$. Its location is $x_4 = a + i_4 = a + 3a + x_0 = \boxed{4a + x_0}$.

$\boxed{3}$. On each passage from one mirror to the other the ray will travel a distance parallel to the mirrors of
   $\Delta x = D \tan \theta$.
   We find the number of reflections before the ray leaves the mirrors from
   $N = L/\Delta x = L/D \tan \theta = (2.0 \text{ m})/(0.10 \text{ m}) \tan 30° = 34.6$,
   so there are 34 reflections. After each reflection the fraction of the intensity remaining is 0.95. After N reflections, we have
   $I/I_0 = (0.95)^N$,
   and the attenuation of the beam is
   $(I_0 - I)/I_0 = 1 - (0.95)^{34} = 0.825$, so 82.5% of the beam has been dissipated.



4. (a) The angle of incidence of the ray from your feet must equal the angle of reflection, so the ray must strike the bottom of the mirror. This will be true if you stand any distance from the mirror.
   (b) The ray from the top of your head must reflect from a point 6 cm below the top of your head to reflect to your eyes. The ray from your feet must reflect from a point 6 cm below where the mirror had been. If you lower the mirror by 6 cm, you will see your full length. Again, this is independent of how close you stand relative to the mirror.

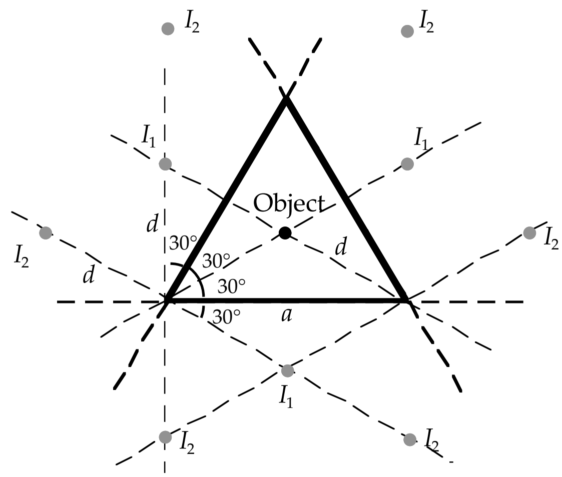
5. We label the three images formed by one reflection I_1. With double reflections, each of these images forms two images from reflections off the other mirrors, which we label I_2. The distance from the axis for the I_1 images is

$$L_1 = 2(\tfrac{1}{2}a)\tan 30° = \boxed{0.577a}.$$

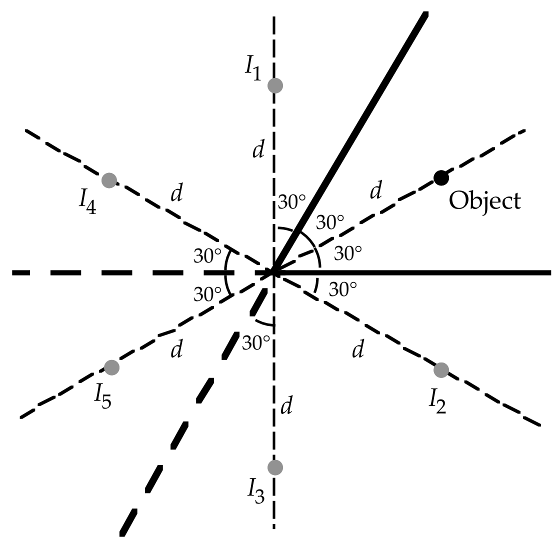
The distance from the axis of the I_2 images is

$$L_2 = 2d\cos 30° = 2(\tfrac{1}{2}a/\cos 30°)\cos 30° = \boxed{a},$$

which can be seen from the figure.



6. Each image is as far behind the mirror as the object is in front of the mirror. Each image is also an object for the other (extended) mirror.



7. We find the focal length of the mirror from

$$\frac{1}{s_1} + \frac{1}{i_1} = \frac{1}{f};$$

$$\frac{1}{+60\text{ cm}} + \frac{1}{+20\text{ cm}} = \frac{1}{f}, \text{ which gives } f = 15\text{ cm}.$$

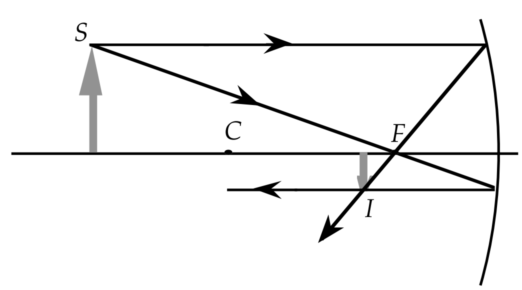
We find the second image position from

$$\frac{1}{s_2} + \frac{1}{i_2} = \frac{1}{f};$$

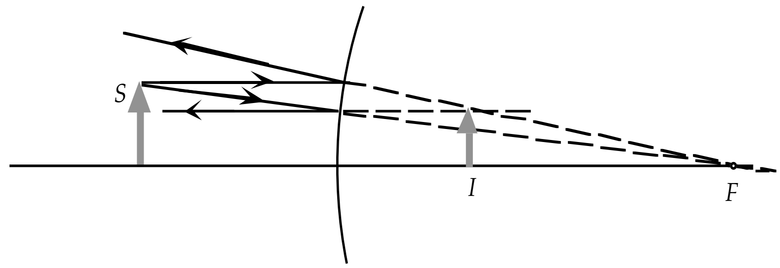
$$\frac{1}{+35\text{ cm}} + \frac{1}{i_2} = \frac{1}{+15\text{ cm}}, \text{ which gives } i_2 = 26.3\text{ cm}.$$

The radius of the mirror is

$$R = 2f = 2(15\text{ cm}) = \boxed{30\text{ cm}}.$$



8. The focal length of the mirror is
$f = R/2 = (-44\text{ cm})/2 = -22$ cm.
We find the location of the image from
$1/s + 1/i = 1/f$;
$1/(28\text{ cm}) + 1/i = 1/(-22\text{ cm})$;
$i = \boxed{-12\text{ cm (behind the mirror)}}$.
We find the magnification from
$M = -i/s$
$= -(-12\text{ cm})/(+28\text{ cm}) = \boxed{0.43}$.



9. The real image must be in front of the mirror.
The height of the image is
$h_i = Mh = (-i/s)h = [-(8.0\text{ cm})/(20\text{ cm})](2.0\text{ cm}) = -0.80$ cm.
The image is $\boxed{\text{0.80 cm tall, inverted, in front of the mirror}}$.

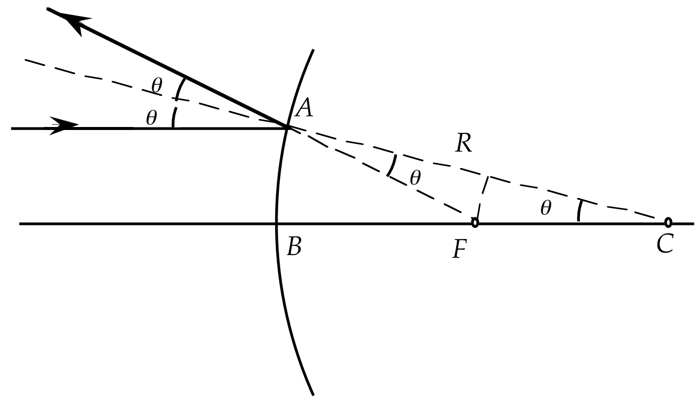
10. The focal length of the mirror is $f = R/2 = (176\text{ cm})/2 = 88$ cm. We find the image distance from
$1/s + 1/i = 1/f$, which gives $i = fs/(s-f)$.
The size of the image is
$h_i = Mh = (-i/s)h = -[f/(s-f)]h = -[(88\text{ cm})/(133\text{ cm} - 88\text{ cm})](6.00\text{ cm}) = \boxed{-11.7\text{ cm}}$.

$\boxed{11}$. We find the location of the image from
$$\frac{1}{s} + \frac{1}{i} = \frac{1}{f} = \frac{2}{R};$$
$$\frac{1}{+80\text{ cm}} + \frac{1}{i} = \frac{2}{+200\text{ cm}}, \text{ which gives } i = -400\text{ cm (behind the mirror).}$$
The height of the image is
$h_i = Mh = (-i/s)h = [-(-400\text{ cm})/(80\text{ cm})](10\text{ cm}) = +50$ cm.

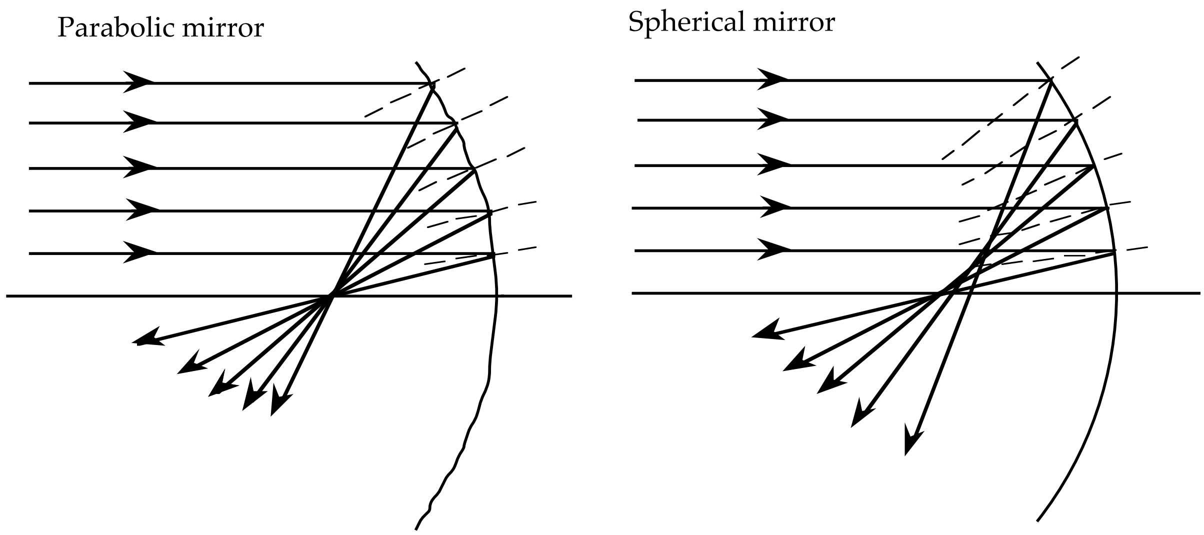
12. The object is a distance $s = x + y$ in front of the convex mirror. Since the image produced by the plane mirror is a distance x behind the plane mirror, it must also be a distance $(x - y)$ behind the convex mirror. We also know that the images produced by both mirrors coincide, so for the convex mirror we have $i = -(x - y)$. Note the negative sign here, as the image is behind the mirror (virtual). So for the convex mirror we have
$1/f = 1/s + 1/i = 1/(x-y) + 1/[-(x-y)] = -2y/(x^2 - y^2)$, which gives
$f = \boxed{-(x^2 - y^2)/2y}$.

13. The distance between the bird and the mirror is s, which satisfies
$1/s + 1/i = 1/f$.
As the bird approaches the mirror at a speed v, s *decreases* at the rate of v:
$v = -ds/dt$.
Take the time derivative of each term in the first equation above:
$d(1/s)/dt + d(1/i)/dt = d(1/f)/dt$;
$-(1/s)^2(ds/dt) - (1/i)^2(di/dt) = 0$;
$(1/s)^2 v - (1/i)^2(di/dt) = 0$;
The velocity of the image is the rate at which the distance between the mirror and the image changes:
$v_i = di/dt = (i/s)^2 v = \boxed{[f/(s-f)]^2 v\text{, away from the mirror}}$,
where in the last step we noted that $i = fs/(s-f)$. The direction of v_i can also be indicated with a negative sign, if one chooses the velocity of the bird as positive.
The bird meets its image when $s = i$, which takes place at $\boxed{s = 2f}$, or two focal lengths from the mirror surface. Note that this is also the center of the sphere of which the mirror is a part.

14. From the diagram, we have
$\angle CAF = \theta$ and $\angle ACF = \theta$.
This means that the triangle ACF is isosceles, with a base of length R.
By dropping a perpendicular from F to the base AC, we have
$CF = (\frac{1}{2}R)/\cos\theta$.
For small θ, $\cos\theta \simeq 1$, so we have
$CF = \frac{1}{2}R$, independent of angle.
The focal length is
$BF = R - (\frac{1}{2}R) = \frac{1}{2}R$.



15.



16. For the refraction from water into glass, we have
$n_1/s + n_2/i = (n_2 - n_1)/R$;
$1.33/(1.5 \text{ cm}) + 1.50/i = (1.50 - 1.33)/(3.0 \text{ cm})$, which gives $i = \boxed{-1.8 \text{ cm}}$.
Because the light passes to the glass, the negative sign means that the image is on the water side, 1.8 cm from the surface of the glass sphere.
Because the rays that form this image are in the glass, this image is $\boxed{\text{virtual}}$.

17. For the refraction at the convex surface, for which $R > 0$, we have
$n_1/s + n_2/i = (n_2 - n_1)/R$;
$(1.0)/(15 \text{ cm}) + (1.5)/i = (1.5 - 1.0)/(10 \text{ cm})$, which gives $i = \boxed{-90 \text{ cm (in air in front of glass)}}$.

18. The ray parallel to the axis can be considered to be from an object at infinity:
$n_1/s + n_2/i = (n_2 - n_1)/R$;
$(1.0)/\infty + n_2/(13 \text{ cm}) = (n_2 - 1.0)/(8.5\text{cm})$, which gives $n_2 = \boxed{2.9}$.

19. Because the glass has parallel sides, we can consider refraction from water to air. The plane surface is treated as a sphere with an infinite radius:
$n_1/s + n_2/i = (n_2 - n_1)/R$;
$(1.33)/(40 \text{ cm}) + (1.0)/i = (1.0 - 1.33)/(\infty)$, which gives $i = \boxed{-30 \text{ cm (behind the glass)}}$.

20. We find the location of the image of the fish from

$$n_1/s_1 + n_2/i_1 = (n_2 - n_1)/R;$$

$$(1.5)/(5\text{ cm}) + (1.0)/i_1 = (1.0 - 1.5)/(-5\text{cm}), \text{ which gives } i_1 = \boxed{-5\text{ cm (behind the surface)}}.$$

This is expected, since the fish is at the center of curvature.
We find the location of the image of the pattern from

$$n_1/s_2 + n_2/i_2 = (n_2 - n_1)/R;$$

$$(1.5)/(10\text{ cm}) + (1.0)/i_2 = (1.0 - 1.5)/(-5\text{cm}), \text{ which gives } i_2 = \boxed{-20\text{ cm (behind the surface)}}.$$

21. We find the location of the image of the fault from

$$n_1/s + n_2/i = (n_2 - n_1)/R;$$

$$(1.6)/(2.3\text{ cm}) + (1.0)/i = (1.0 - 1.6)/(-0.8\text{ cm}), \text{ which gives } i = \boxed{+18.4\text{ cm}}.$$

This image is 18.4 cm in front of the glass. If we assume that a person can see clearly objects that are at least 25 cm away, you should be at least $\boxed{\text{43 cm from the surface}}$.

22. For the refraction at the convex surface, for which $R > 0$, we have

$$n_1/s + n_2/i = (n_2 - n_1)/R, \text{ which gives}$$

$$1/i = (n_2 - n_1)/n_2R - n_1/n_2s.$$

For a virtual image, $i < 0$, or

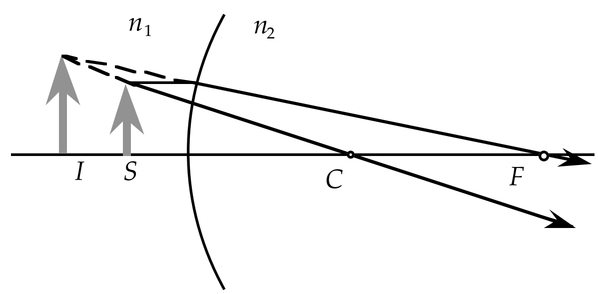
$$n_1/n_2s > (n_2 - n_1)/n_2R, \text{ which gives}$$

$$s_c = \boxed{n_1R/(n_2 - n_1)}.$$

For the ray tracing, we use

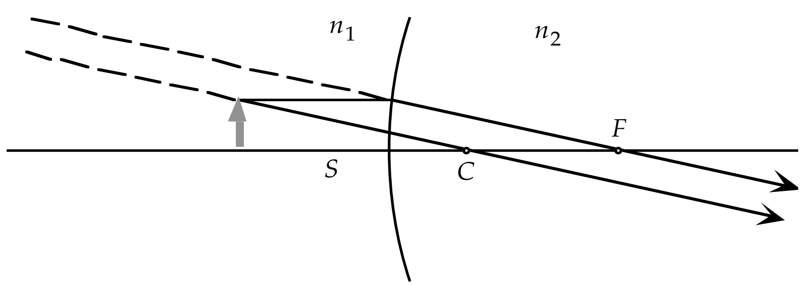
$$f = n_2R/(n_2 - n_1) > R.$$

We see that the image is upright and magnified.

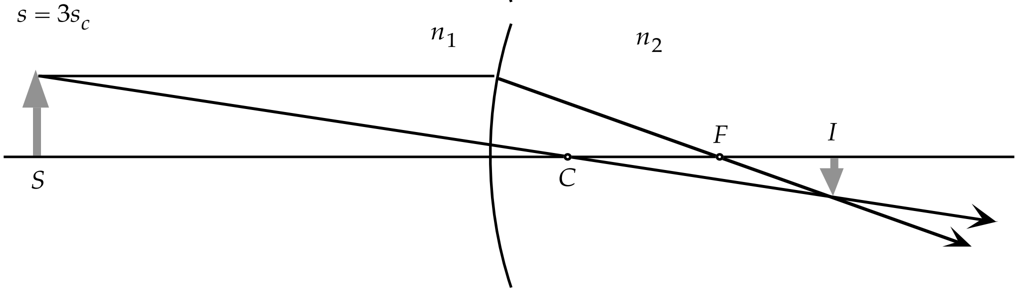


23.

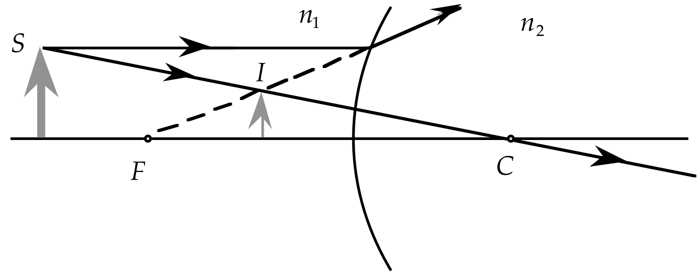
$s = s_c$



$s = 3s_c$



24. For the ray tracing, we use $f = n_2R/(n_2 - n_1) < 0$.
$\boxed{\text{The image is upright, virtual, and reduced}}$.
For the refraction at the convex surface, for which $R > 0$, we have
$n_1/s + n_2/i = (n_2 - n_1)/R$, which gives
$1/i = (n_2 - n_1)/n_2R - n_1/n_2s$.
Because $n_2 < n_1$, both terms are negative, so $i < 0$ for all positive values of s.
There is $\boxed{\text{no critical distance}}$ at which the nature of the image changes.



25. (a) For the refraction at the convex surface, for which $R > 0$, we have
$n_1/s + n_2/i = (n_2 - n_1)/R$.
For an object very far from the surface, $s \to \infty$, which gives
$0 + n_2/i = (n_2 - n_1)/R$, or $i = n_2R/(n_2 - n_1)$.
Because $n_2 > n_1$, $i > 0$, the image is real. The magnification is
$M = -i/s = -n_2R/(n_2 - n_1)s$.
Because $n_2 > n_1$, $M < 0$; the image is inverted.
Because s is very large, the image is reduced.

(b) For the refraction at the convex surface, for which $R > 0$, we have
$n_1/s + n_2/i = (n_2 - n_1)/R$.
For an image very far from the surface, $i \to \infty$, which gives
$n_1/s + 0 = (n_2 - n_1)/R$, or
$s = \boxed{n_1R/(n_2 - n_1)}$.

(c) If the object position is δ less than the critical value from part (b), we have
$s = n_1R/(n_2 - n_1) - \delta = (n_1R - x)/(n_2 - n_1)$, where $x = (n_2 - n_1)\delta$.
For the refraction at the convex surface, we have
$n_1/s + n_2/i = (n_2 - n_1)/R$;
$n_1(n_2 - n_1)/(n_1R - x) + n_2/i = (n_2 - n_1)/R$, which reduces to
$i = n_2R(n_1R - x)/(n_2 - n_1)(-x)$.
When x is small, we have
$i \simeq -n_2n_1R^2/(n_2 - n_1)x = -n_2n_1R^2/(n_2 - n_1)^2\delta$.
When $\delta \to 0$, $i \to -\infty$. $\boxed{\text{The image is very far in front of the boundary}}$.
Because the image is on the side opposite to the outgoing light, the image is virtual.

(d) As s decreases to 0, $\delta \to n_1R/(n_2 - n_1)$, and $x \to n_1R$. Thus, $i \to 0$.
$\boxed{\text{The position of the image approaches the boundary}}$.

26. For the refraction at the concave surface, for which $R < 0$, we have
$n_1/s + n_2/i = (n_2 - n_1)/R$;
$n_1/s + n_2/(-s) = (n_2 - n_1)/R$, which gives
$\boxed{s = -R}$.
Note that $R < 0$, so $s > 0$, a real object.

27. From the diagram, we have

$\theta_1 = \theta_2 + \alpha,$

$h/R = \sin\ \theta_1,$

$h/f \approx \sin\ \alpha.$

For the refraction at the surface, we have

$n_1 \sin \theta_1 = n_2 \sin \theta_2 .$

For small angles, this becomes

$n_1\theta_1 \approx n_2\theta_2$; so

$h/R = \sin\ \theta_1 \approx \theta_1 = \theta_2 + \alpha$

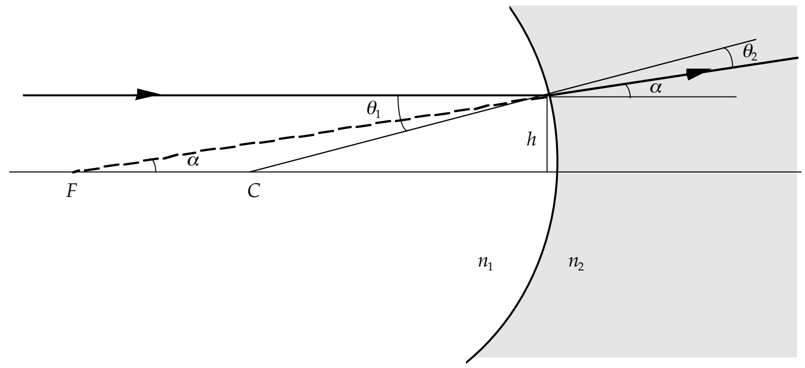
$\approx (n_1/n_2)\ \theta_1 + \alpha$

$\approx (n_1/n_2)\ \sin\ \theta_1 + \sin\ \alpha$

$= (n_1/n_2)(h/R) + h/f\ ;$

$h/f \approx\ (1 - n_1/n_2)(h/R);$ which gives

$f\ \approx R/\ (1 - n_1/n_2)\ = [n_2\ /(n_2 - n_1)]R.$



28. Look at Fig. 36-45. In the triangle *ACI* we have

$\theta_2\ + \alpha + (\pi\ - \beta) = \pi$, so $\ \theta_2\ =\ \beta - \alpha.$

Similarly, in the triangle *ACS* we have

$\beta + \gamma + (\pi\ - \theta_1) = \pi$, so $\theta_1\ =\ \beta + \gamma.$

For the refraction at the surface, we have

$n_1 \sin \theta_1 = n_2 \sin \theta_2$;

$n_1 \sin (\beta + \gamma) = n_2 \sin (\beta - \alpha).$

For small angles, this becomes

$n_1 (\beta + \gamma) \approx n_2 (\beta - \alpha).$

For small angles, the arc *AB* is nearly a straight line that's perpendicular to the line connecting *S* and *I*; so

$AB/s \approx \tan\ \gamma\ \approx \gamma$; $AB/R \approx \tan\ \beta \approx \beta$; $\ AB/i \approx \sin\ \alpha \approx \alpha$.

Plug these into $n_1 (\beta + \gamma) \approx n_2 (\beta - \alpha)$ to obtain

$n_1 (AB/R\ + AB/s) \approx n_2 (AB/R\ - AB/i);$

$n_1 (1/R\ + 1/s) \approx n_2 (1/R\ - 1/i),$ which leads to Eq. 36-10:

$n_1\ /s\ +\ n_2\ /i\ =\ (n_2 - n_1)/R$.

29. From the diagram, we have

$\theta_2 = \beta + \alpha,\ \theta_1 = \beta + \gamma,$

$AB = BC \tan\ \beta = SB \tan\ \gamma = IB \tan\ \alpha,$ where

$BC = R,\ SB = s,$ and

$IB = -\,i$ (from the sign convention).

When the angles are small, $\sin\ \phi \simeq \tan\ \phi \simeq \phi,$ so we have

$AB = R\ \beta = s\ \gamma = -\ i\ \alpha.$

For the refraction at the surface, we have

$n_1 \sin \theta_1 = n_2 \sin \theta_2$.

For small angles, this becomes

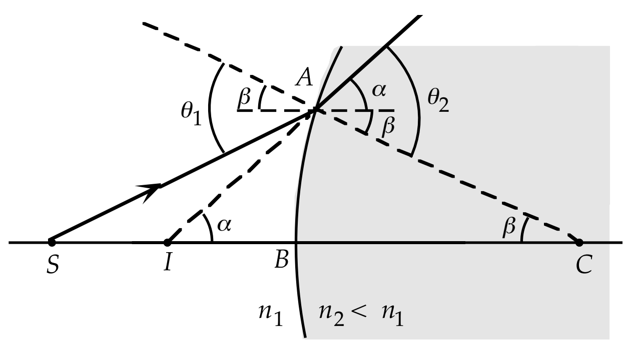
$n_1\theta_1 = n_2\theta_2$;

$n_1(\beta + \gamma) = n_2(\beta + \alpha);$

$n_1\ (AB/R + AB/s) = n_2\ (AB/R - AB/i),$

which gives

$n_1/s\ +\ n_2/i\ =\ (n_2 - n_1)/R.$



30. From the diagram, we have

$\theta_2 = \beta - \alpha$, $\theta_1 = \beta - \gamma$,

$AB = BC \tan\beta = SB \tan\gamma$

$= IB \tan\alpha$, where

$BC = -R$, $SB = s$, and $IB = -i$

(from the sign convention).

When the angles are small,

$\sin\phi \simeq \tan\phi \simeq \phi$, so we have

$AB = -R\,\beta = s\,\gamma = -i\,\alpha$.

For the refraction at the surface, we have

$n_1 \sin\theta_1 = n_2 \sin\theta_2$.

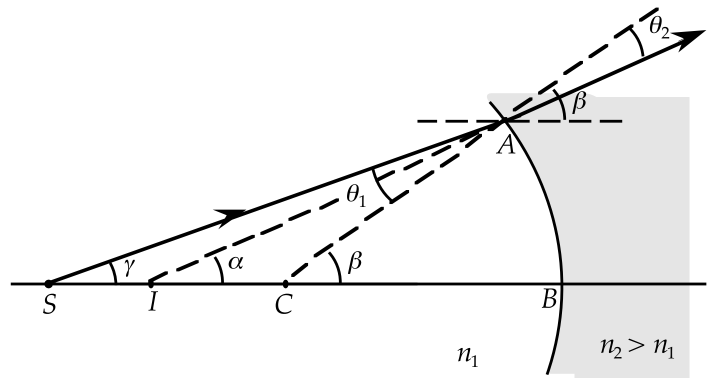
For small angles, this becomes

$n_1\theta_1 = n_2\theta_2$;

$n_1(\beta - \gamma) = n_2(\beta - \alpha)$;

$n_1(-AB/R - AB/s) = n_2\,[-AB/R - (-AB/i)]$, which gives

$n_1/s + n_2/i = (n_2 - n_1)/R$.



31. (a) For the refraction of a thin lens, we have

$$\frac{1}{s} + \frac{1}{i} = \frac{1}{f};$$

$$\frac{1}{+24\text{ cm}} + \frac{1}{+51\text{ cm}} = \frac{1}{f},\ \text{which gives}\ f = +16\text{ cm}.$$

(b) Because the focal length is positive, the lens is $\boxed{\text{converging}}$.

(c) Because the image is on the side of the outgoing light ($i > 0$), the image is $\boxed{\text{real}}$.

The magnification is

$$M = -\frac{i}{s} = -\frac{+51\text{ cm}}{+24\text{ cm}} = -2.1.$$

Because the magnification is negative, the image is $\boxed{\text{inverted}}$.

(d) The magnification is $\boxed{-2.1}$.

32. The first surface has a negative radius, and the second surface has a positive radius. The focal length of the lens in air satisfies

$1/f = [(n-1)/1](1/R_1 - 1/R_2)$

$= (1.56 - 1)[\,1/(-6.0\text{ cm}) - 1/(7.5\text{ cm})]$, which gives $f = \boxed{-6.0\text{ cm}}$.

$\boxed{33}$. (a) For the refraction of the thin lens, we have

$$\frac{1}{s} + \frac{1}{i} = \frac{1}{f};$$

$$\frac{1}{+15\text{ cm}} + \frac{1}{i} = \frac{1}{-22\text{ cm}},\ \text{which gives}\quad i = -8.9\text{ cm}.$$

The image is $\boxed{\text{8.9 cm in front of the lens}}$.

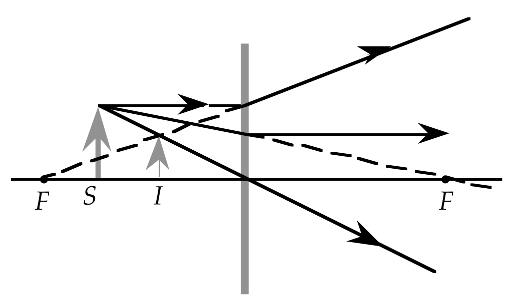
(b) $\boxed{\text{No}}$, because $i < 0$, the image is virtual.

(c) The magnification is

$$M = -\frac{i}{s} = -\frac{-8.9\text{ cm}}{+15\text{ cm}} = +0.60.$$

$\boxed{\text{Yes}}$, because $M > 0$, the image is upright.

(d) The magnification is $\boxed{+0.60}$.

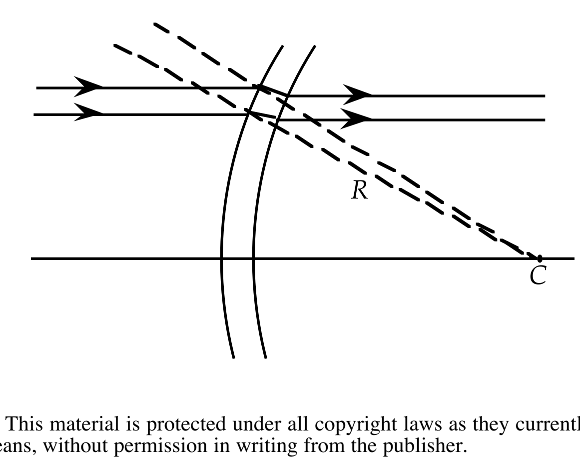


34. We find the image distance of a real source, $s > 0$, for a thin lens from
$1/s + 1/i = 1/f$, which gives $i = fs/(s - f)$.
For a real image, $i > 0$.
If we have a negative lens, $f < 0$. Thus $(s - f) > 0$, and it is not possible to have a real image.
If we have a positive lens, $f > 0$. Thus we need $(s - f) > 0$. The conditions are $\boxed{f > 0, \text{ and } s > f}$.

35. (a) For a real image, $s > 0$, and $i > 0$. The magnification is
$M = -i/s = -2$, which gives $i = 2s$.
For the refraction of the thin lens, we have
$\frac{1}{s} + \frac{1}{i} = \frac{1}{f}$;
$\frac{1}{s} + \frac{1}{2s} = \frac{1}{+25 \text{ cm}}$, which gives $s = +38$ cm.
(b) For a virtual image, $s > 0$, and $i < 0$. The magnification is
$M = -i/s = +2$, which gives $i = -2s$.
For the refraction of the thin lens, we have
$\frac{1}{s} + \frac{1}{i} = \frac{1}{f}$;
$\frac{1}{s} + \frac{1}{-2s} = \frac{1}{+25 \text{ cm}}$, which gives $s = +13$ cm.

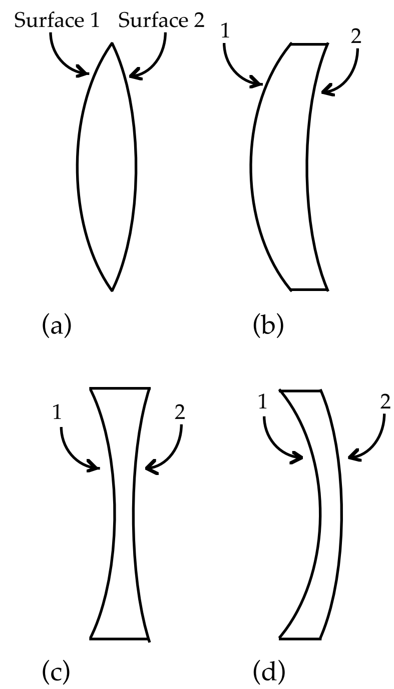
36. For the refraction of the thin lens, we have
$\frac{1}{s} + \frac{1}{i} = \frac{1}{f}$, and $h_i = Mh = -ih/s$.
(a) For $s = 50$ cm, we have
$\frac{1}{+50 \text{ cm}} + \frac{1}{i} = \frac{1}{+25 \text{ cm}}$, which gives $i = +50$ cm.
$h_i = -ih/s = -(+50 \text{ cm})(3 \text{ cm})/(+50 \text{ cm}) = -3$ cm.
The image is 50 cm behind the lens (real), 3 cm high, and inverted.
(b) For $s = 30$ cm, we have
$\frac{1}{+30 \text{ cm}} + \frac{1}{i} = \frac{1}{+25 \text{ cm}}$, which gives $i = +150$ cm.
$h_i = -ih/s = -(+150 \text{ cm})(3 \text{ cm})/(+30 \text{ cm}) = -15$ cm.
The image is 150 cm behind the lens (real), 15 cm high, and inverted.
(c) For $s = 20$ cm, we have
$\frac{1}{+20 \text{ cm}} + \frac{1}{i} = \frac{1}{+25 \text{ cm}}$, which gives $i = -100$ cm.
$h_i = -ih/s = -(-100 \text{ cm})(3 \text{ cm})/(+20 \text{ cm}) = +15$ cm.
The image is 100 cm in front of the lens (virtual), 15 cm high, and upright.
(d) For $s = 5$ cm, we have
$\frac{1}{+5 \text{ cm}} + \frac{1}{i} = \frac{1}{+25 \text{ cm}}$, which gives $i = -6.25$ cm.
$h_i = -ih/s = -(-6.25 \text{ cm})(3 \text{ cm})/(+5 \text{ cm}) = +3.75$ cm.
The image is 6.25 cm in front of the lens (virtual), 3.75 cm high, and upright.

37. For a thin lens, the centers of the two spherical surfaces are at the same location. The surfaces for the two refractions of a ray are parallel, so the ray is displaced but undeviated. Two parallel rays will still be parallel after passing through the lens. Thus the focal length is infinite.
From the refraction of the thin lens, we have
$i = -s$,
which means that a real object produces a $\boxed{\text{virtual image}}$.
Note that this is as if the lens were not there.



38. (a) For the distant object, the image distance is the focal length: $f = \boxed{+38\text{ cm}}$.
(b) For the refraction of the thin lens, we have
$$1/s_a + 1/i = 1/f;$$
$$1/(75\text{ cm}) + 1/i = 1/(38\text{ cm}),\ \text{which gives } i = \boxed{77\text{cm}},$$
The magnification is $M = -\ i/s = -(77\text{cm})/(75\text{ cm}) = \boxed{-1.03}$.
The image has a magnification of – 1.03, is 77 cm behind the lens, real, and inverted.
(c) The focal length of the lens in air is given by
$$1/f_1 = [(n-1)/1](1/R_1 - 1/R_2).$$
The focal length of the lens in the liquid is given by
$$1/f_2 = [(n-n_{\text{liquid}})/n_{\text{liquid}}](1/R_1 - 1/R_2).$$
If we divide the two equations, we get
$$f_2/f_1 = (n-1)n_{\text{liquid}}/(n-n_{\text{liquid}});$$
$$f_2/(+\ 38\text{ cm}) = (1.55-1)(1.33)/(1.55-1.33),\ \text{which gives } f_2 = \boxed{+126\text{ cm}}.$$

39. The image of a distant source will be at the focal point, so the image distance will be the focal length, which we find from
$$1/f = (n-1)(1/R_1 - 1/R_2).$$



Lens *a*:
(a) $1/f_a = (1.55-1)[1/(+\ 25\text{ cm}) - 1/(-\ 60\text{ cm})]$, or $f_a = \boxed{32.1\text{ cm}}$.
The image will be
$$i = f_a s/(s-f_a) = (32.1\text{ m})(10\text{ m})/(10\text{ m} - 0.321\text{ m})$$
$= \boxed{\text{33 cm beyond the lens}}$.
(b) The positive image distance means that the image is $\boxed{\text{inverted and real}}$.
(c) $M = -i/s = -(+)/(+) = \boxed{-}$, which is consistent with part (b).

Lens *b*:
(a) $1/f_b = (1.55-1)[1/(+\ 25\text{ cm}) - 1/(+\ 60\text{ cm})]$,
which gives $f_b = \boxed{+77.9\text{ cm}}$.
The image will be
$$i = f_b s/(s-f_b) = (77.9\text{ cm})(10\text{ m})/(10\text{ m} - 0.779\text{ m})$$
$= \boxed{\text{84 cm beyond the lens}}$.
(b) The positive image distance means that the image is $\boxed{\text{inverted and real}}$.
(c) $M = -i/s = -(+)/(+) = \boxed{-}$, which is consistent with part (b).

Lens *c*:
(a) $1/f_c = (1.55-1)[1/(-\ 25\text{ cm}) - 1/(+\ 60\text{ cm})]$,
which gives $\boxed{f_c = -\ 32.1\text{ cm}}$.
The image will be
$$i = f_c s/(s-f_c) = (-32.1\text{ cm})(10\text{ m})/[10\text{ m} - (-\ 0.321\text{ m})]$$
$= -31$ cm, or $\boxed{\text{31 cm in front of the lens}}$.
(b) The negative image distance means that the image is $\boxed{\text{upright and virtual}}$.
(c) $M = -i/s = -(-)/(+) = \boxed{+}$, which is consistent with part (b).

Lens *d*:
(a) $1/f_d = (1.55-1)[1/(-\ 25\text{ cm}) - 1/(-\ 60\text{ cm})]$, which gives $f_d = \boxed{-77.9\text{ cm}}$.
The image will be
$$i = f_d s/(s-f_d) = (-\ 77.9\text{ cm})(10\text{ m})/[10\text{ m} - (-\ 0.779\text{ m})] = -\ 73\text{ cm},$$ or $\boxed{\text{73 cm in front of the lens}}$.
(b) The negative image distance means that the image is $\boxed{\text{upright and virtual}}$.
(c) $M = -i/s = -(-)/(+) = \boxed{+}$, which is consistent with part (b).

40. The focal lengths will be the same as those found in Problem 39.

Lens *a*:

We locate the image from

$1/s_a + 1/i_a = 1/f_a$; $1/(25 \text{ cm}) + 1/i_a = 1/(+32.1 \text{ cm})$, which gives $i_a = -113$ cm.

The magnification is

$M_a = -i_a/s_a = -(-113 \text{ cm})/(25 \text{ cm}) = +4.5$.

$\boxed{\text{The image is 113 cm to the right of the lens, upright, virtual, with } M = +4.5}$.

Lens *b*:

We locate the image from

$1/s_b + 1/i_b = 1/f_b$; $1/(25 \text{ cm}) + 1/i_b = 1/(+77.9 \text{ cm})$, which gives $i_b = -36.8$ cm.

The magnification is

$M_b = -i_b/s_b = -(-36.8\text{cm})/(25 \text{ cm}) = +1.5$.

$\boxed{\text{The image is 36.8 cm to the right of the lens, upright, virtual, with } M = +1.5}$.

Lens *c*:

We locate the image from

$1/s_c + 1/i_c = 1/f_c$; $1/(25 \text{ cm}) + 1/i_c = 1/(-32.1 \text{ cm})$, which gives $i_c = -14.1$ cm.

The magnification is

$M_c = -i_c/s_c = -(-14.1 \text{ cm})/(25 \text{ cm}) = +0.56$.

$\boxed{\text{The image is 14.1 cm to the right of the lens, upright, virtual, with } M = +0.56}$.

Lens *d*:

We locate the image from

$1/s_d + 1/i_d = 1/f_d$; $1/(25 \text{ cm}) + 1/i_d = 1/(-77.9 \text{ cm})$, which gives $i_d = -18.9$ cm.

The magnification is

$M_d = -i_d/s_d = -(-18.9 \text{ cm})/(25 \text{ cm}) = +0.76$.

$\boxed{\text{The image is 18.9 cm to the right of the lens, upright, virtual, with } M = +0.76}$.

41. Lens *a*:

We locate the image from

$1/s_a + 1/i_a = 1/f_a$; $1/(65 \text{ cm}) + 1/i_a = 1/(+32.1 \text{ cm})$, which gives $i_a = +63.4$ cm.

The magnification is

$M_a = -i_a/s_a = -(+63.4)/(65 \text{ cm}) = -0.98$.

$\boxed{\text{The image is 63.4 cm to the left of the lens, inverted, real, with } M = -0.98}$.

Lens *b*:

We locate the image from

$1/s_b + 1/i_b = 1/f_b$; $1/(65 \text{ cm}) + 1/i_b = 1/(+77.9 \text{ cm})$, which gives $i_b = -393$ cm.

The magnification is

$M_b = -i_b/s_b = -(-393 \text{ cm})/(65 \text{ cm}) = +6.0$.

$\boxed{\text{The image is 393 cm to the right of the lens, upright, virtual, with } M = +6.0}$.

Lens *c*:

We locate the image from

$1/s_c + 1/i_c = 1/f_c$; $1/(65 \text{ cm}) + 1/i_c = 1/(-32.1 \text{ cm})$, which gives $i_c = -21.5$ cm.

The magnification is

$M_c = -i_c/s_c = -(-21.5 \text{ cm})/(65 \text{ cm}) = +0.33$.

$\boxed{\text{The image is 21.5 cm to the right of the lens, upright, virtual, with } M = +0.33}$.

Lens *d*:

We locate the image from

$1/s_d + 1/i_d = 1/f_d$; $1/(65 \text{ cm}) + 1/i_d = 1/(-77.9 \text{ cm})$, which gives $i_d = -35.4$ cm.

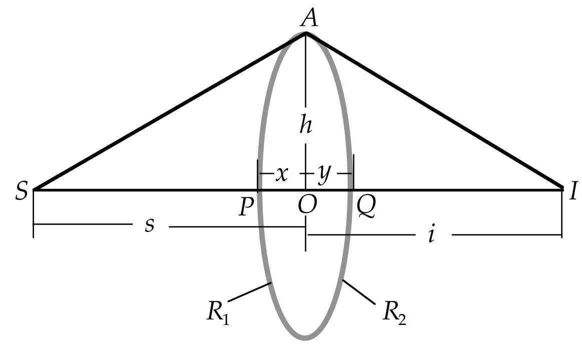
The magnification is

$M_d = -i_d/s_d = -(-35.4 \text{ cm})/(65 \text{ cm}) = +0.55$.

$\boxed{\text{The image is 35.4 cm to the right of the lens, upright, virtual, with } M = +0.55}$.

42. It is clear form the problem statement that
    $s = X_0 + f$ and $i = X_1 + f$.
    Plug these into the thin lens equation:
    $1/s + 1/i = 1/f$;
    $1/(X_0 + f) + 1/(X_1 + f) = 1/f$;
    $(X_0 + f)(X_1 + f) = f(X_0 + X_1 + 2f)$;
    $X_0 X_1 + fX_0 + fX_1 + f^2 = fX_0 + fX_1 + 2f^2$, which gives $X_0 X_1 = f^2$.

43. First, let's calculate the time t_1 it take for light to travel along the path SAI:
    $t_1 = SA/c + AI/c$
    $= (s^2 + h^2)^{1/2}/c + (i^2 + h^2)^{1/2}/c$
    $\approx (s/c)(1 + h^2/2s^2) + (i/c)(1 + h^2/2i^2)$
    $= s/c + h^2/2sc + i/c + h^2/2ic$.
    Here we used the small-angle approximation: $h << s$, $h << i$.
    Next, calculate the time t_2 it takes for light to travel along the direct path $SPOQI$:
    $t_2 = SP/c + PQ/(c/n) + QI/c$
    $= (s - x)/c + n(x + y)/c + (i - y)/c$
    $= s/c + i/c + (n - 1)(x + y)/c$.



From the hint given in the problem statement we have
    $h^2 = x(2R_1 - x) \approx 2R_1 x$ and $h^2 = y(2R_2 - y) \approx 2R_2 y$, which gives
    $x = h^2/2R_1$ and $y = h^2/2R_2$.
    Equate t_1 and t_2:
    $t_1 = t_2$:
    $s/c + h^2/2sc + i/c + h^2/2ic = s/c + i/c + (n - 1)(x + y)/c$;
    $h^2/2sc + h^2/2ic = (n - 1)(x + y)/c = (n - 1)(h^2/2R_1 + h^2/2R_2)/c$, which gives
    $1/s + 1/i = (n - 1)(1/R_1 + 1/R_2)$.

44. We find the focal length by finding the image distance for an object very far away.
    For the first lens, we have
    $1/s_1 + 1/i_1 = 1/f_1$;
    $0 + 1/i_1 = 1/f_1$, or, as expected, $i_1 = f_1$.
    The first image is the object for the second lens. If the first image is real, the second object is virtual:
    $s_2 = - i_1 = - f_1$.
    For the second lens, we have
    $1/s_2 + 1/i_2 = 1/f_2$;
    $1/(- f_1) + 1/i_2 = 1/f_2$.
    Because the second image must be at the focal point of the combination, we have
    $- 1/f_1 + 1/f = 1/f_2$, which gives
    $1/f = 1/f_1 + 1/f_2$.

$\boxed{45}$. When the object (the book) is 30 cm from the lens, we want the image to be at the near point, which is a virtual image 70 cm in front of the lens. From the refraction for a thin lens, we have
    $1/s + 1/i = 1/f$;
    $1/(30 \text{ cm}) + 1/(- 70 \text{ cm}) = 1/f$, which gives $f = \boxed{+53 \text{ cm}}$.

46. (a) When the object is far from the lens, we want the image to be at the far point, which is a virtual image 41 cm in front of the lens. From the refraction for a thin lens, we have

$1/s + 1/i = 1/f;$

$1/\infty + 1/(-41 \text{ cm}) = 1/f$, which gives $f = -41$ cm.

$\boxed{\text{A diverging lens with } f = -41 \text{ cm is needed}}$.

(b) We find the corrected near point by finding the object distance that produces a virtual image at the near point, 14 cm in front of the lens. From the refraction for a thin lens, we have

$1/s + 1/i = 1/f;$

$1/s + 1/(-12 \text{ cm}) = 1/(-41 \text{ cm})$, which gives $f = -17$ cm.

$\boxed{\text{The corrected near point is 17 cm, which means the book is easily readable}}$.

47. We find the image distance for a thin lens from

$1/s + 1/i = 1/f$, which gives $i = fs/(s - f)$.

For the magnification we have

$M = -i/s = -f/(s - f);$

$3 = -(9 \text{ cm})/(s - 9 \text{ cm})$, which gives $s = \boxed{+6 \text{ cm}}$.

Note that the image distance is – 18 cm, so the image is virtual.

48. We find the magnification of the telescope from

$M = f_1/f_2 = (80 \text{ cm})/(1.7 \text{ cm}) = \boxed{47}$.

49. We find the separation of the lenses from the magnification:

$M = Ld_{\min}/f_1 f_2;$

$60 = L(25 \text{ cm})/(1.0 \text{ cm})(4.0 \text{ cm})$, which gives $L = \boxed{9.6 \text{ cm}}$.

50. The magnification of the telescope is given by

$M = f_1/f_2 = 120.$

For both object and image far away, the separation of the lenses is

$L = i_1 + s_2 = f_1 + f_2;$

$70 \text{ cm} = 120 + f_2$, which gives $\boxed{f_1 = 69.4 \text{ cm}, \ f_2 = 0.58 \text{ cm}}$.

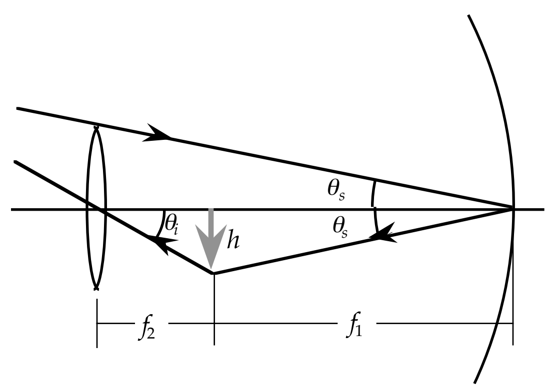
51. Because all parallel rays will form an image at the same point, we consider the ray that reflects from the center of the mirror and the ray that passes through the center of the lens.

The angular magnification is

$M = \theta_i/\theta_s \simeq (\tan \theta_i)/(\tan \theta_s)$

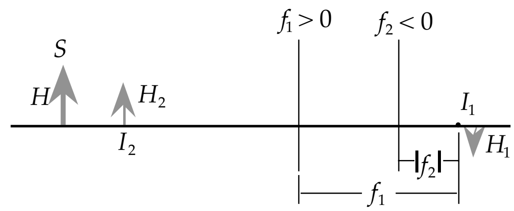
$= (h/f_2)/(h/f_1) = \boxed{f_1/f_2}$.

Note that this is the same as the magnification of the refracting telescope. The image will be inverted.



52. The focal length of the objective lens f_1 is positive. We put an object of height h a distance s_1 from the objective lens. We find the image formed by the objective lens from

$1/s_1 + 1/i_1 = 1/f_1$, which gives $i_1 = f_1 s_1/(s_1 - f_1) > 0$.



The height of this image is

$H_1 = -H(i_1/s_1)$, inverted.

This image is a virtual object, negative object distance, for the eyepiece. The focal length of the eyepiece lens f_2 is negative. The object distance is

$s_2 = -[i_1 - (f_1 + f_2)] = f_1 + f_2 - i_1 = f_1 + f_2 - f_1 s_1/(s_1 - f_1) = [s_1 f_2 - f_1(f_1 + f_2)]/(s_1 - f_1) < 0$.

We find the image formed by the eyepiece lens from

$1/s_2 + 1/i_2 = 1/f_2$, which gives $i_2 = f_2 s_2/(s_2 - f_2)$. Because $f_2 < 0$, $s_2 < 0$, $s_2 - f_2 < 0$; $i_2 < 0$.

The height of this image is

$H_2 = -H_1(i_2/s_2) = +H(i_1/s_1)(i_2/s_2)$, erect.

Because the object and final image are distant, for the subtended angle we can use the tangent. The angular size of the object is

$\theta = H/s_1$.

Because the final image is erect and $i_2 < 0$, the angular size of the image is

$\theta_2 = -H_2/i_2 = -H(i_1/s_1 s_2) = -\theta i_1/s_2$.

The angular magnification is

$$\begin{aligned} M_\theta &= \theta_2/\theta = -i_1/s_2 \\ &= -[f_1 s_1/(s_1 - f_1)]/\{[s_1 f_2 - f_1(f_1 + f_2)]/(s_1 - f_1)\} \\ &= \boxed{f_1 s_1/[f_1(f_1 + f_2) - s_1 f_2]}. \end{aligned}$$

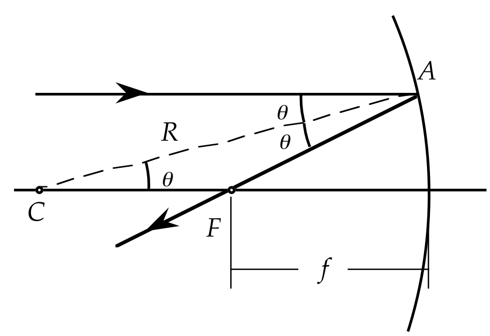
If $s_1 \to \infty$,

$\boxed{M_\theta \to f_1 s_1/(-s_1 f_2) = -f_1/f_2, > 0}$.

An advantage of a Galilean telescope is that it shows an erect image.

53. From the diagram, we have

$\angle CAF = \theta$ and $\angle ACF = \theta$.



This means that the triangle ACF is isosceles, with a base of length R. By dropping a perpendicular from F to the base AC, we have

$CF = (\frac{1}{2}R)/\cos\theta$.

The distance from the mirror to the point F is

$f = R - CF = R - (\frac{1}{2}R)/\cos\theta$, which gives

$f = R[1 - 1/(2\cos\theta)]$.

For small θ, $\cos\theta \simeq 1$, so we have

$f = R[1 - \frac{1}{2}] = \frac{1}{2}R$.

54. If ℓ is the arc length, the maximum angle that a radial line from the point C makes with the axis is

$\theta_{max} = \frac{1}{2}\ell/R = \frac{1}{2}(46\text{ cm})/(18\text{ cm}) = 1.28\text{ rad} = \boxed{73.2°}$.

The maximum value for f occurs when $\theta = 0°$:

$f_{max} = \frac{1}{2}R = \frac{1}{2}(18\text{ c m}) = \boxed{9.0\text{ cm}}$.

The minimum value of f occurs when $\theta = \theta_{max}$:

$f_{min} = R[1 - 1/(2\cos\theta_{max})] = (18\text{ cm})[1 - 1/(2\cos 73.2°)] = \boxed{-13\text{ cm}}$.

The spread in the values of f is

$\Delta f = f_{max} - f_{min} = 9.0\text{ cm} - (-13\text{ cm}) = \boxed{24\text{ cm}}$.

55. We could calculate the focal lengths directly; however, the changes in index are small, so we can use differentials. We differentiate the direct expression for the focal length:

$$f = \frac{1}{(n-1)}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)^{-1};$$

$$\mathrm{d}f = -\frac{\mathrm{d}n}{(n-1)^2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)^{-1}.$$

If we use the average index, we get

$$\mathrm{d}f = -\frac{(1.48135 - 1.48523)}{(1.48329 - 1)^2}\left(\frac{1}{20.00\text{ cm}} - \frac{1}{28.75\text{ cm}}\right)^{-1} = 1.09\text{ cm}.$$

56. Because $f << d$, all of the rays can be considered to be paraxial, so they will all be focused at the focal point. At a distance L from the mirror, the circular cross-section of the cone of light reflecting from the mirror has a radius r. This point is a distance $f - L$ from the focal point, so we can relate the diameter of the spot at that point to the diameter of the mirror:

$$2r/d = (f - L)/f.$$

The area of the spot is

$$A = \pi r^2 = \pi[(f - L)d/2f]^2 = \boxed{[(f - L)/f]^2 \pi d^2/4}.$$

Because all of the light that reflects from the mirror must pass through the cone, we have

$$I_L A = I A_{\text{mirror}};$$

$I_L[(f - L)/f]^2 \pi d^2/4 = I\pi d^2/4$, which gives

$$I_L = \boxed{[f/(f - L)]^2 I}.$$

57. We find the focal lengths of the lenses from

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right);$$

$\frac{1}{f_1} = (1.50 - 1)\left(\frac{1}{+35\text{ cm}} - \frac{1}{-35\text{ cm}}\right)$, which gives $f_1 = +35$ cm;

$\frac{1}{f_2} = (1.50 - 1)\left(\frac{1}{-35\text{ cm}} - \frac{1}{+35\text{ cm}}\right)$, which gives $f_2 = -35$ cm.

If the object is near the positive lens, for the two refractions, we have

$$\frac{1}{s_1} + \frac{1}{i_1} = \frac{1}{f_1};$$

$\frac{1}{+10\text{ cm}} + \frac{1}{i_1} = \frac{1}{+35\text{ cm}}$, which gives $i_1 = -14$ cm;

$$\frac{1}{s_2} + \frac{1}{i_2} = \frac{1}{f_2};$$

$\frac{1}{+29\text{ cm}} + \frac{1}{i_2} = \frac{1}{-35\text{ cm}}$, which gives $i_2 = -15.8$ cm.

The image is 15.8 cm in front of the negative lens, or 0.8 cm from the positive lens on the object side.

If the object is near the negative lens, for the two refractions, we have

$$\frac{1}{s_3} + \frac{1}{i_3} = \frac{1}{f_3};$$

$\frac{1}{+10\text{ cm}} + \frac{1}{i_3} = \frac{1}{-35\text{ cm}}$, which gives $i_3 = -7.78$ cm;

$$\frac{1}{s_4} + \frac{1}{i_4} = \frac{1}{f_4};$$

$\frac{1}{+22.8\text{ cm}} + \frac{1}{i_4} = \frac{1}{+35\text{ cm}}$, which gives $i_4 = -65.4$ cm.

The image is 65.4 cm in front of the positive lens, or 50.4 cm from the negative lens on the object side. The order of the lenses is important.

58. (a) For the first refraction, we have

$n_1/s_1 + n_2/i_1 = (n_2 - n_1)/R_1$;

$1.0/(12 \text{ cm}) + 1.6/i_1 = (1.6 - 1.0)/(18\text{cm})$, which gives $i_1 = -32\text{cm}$.

The first image is 32 cm in front of surface 1.

(b) For the second refraction, we have

$n_2/s_2 + n_1/i_2 = (n_1 - n_2)/R_2$;

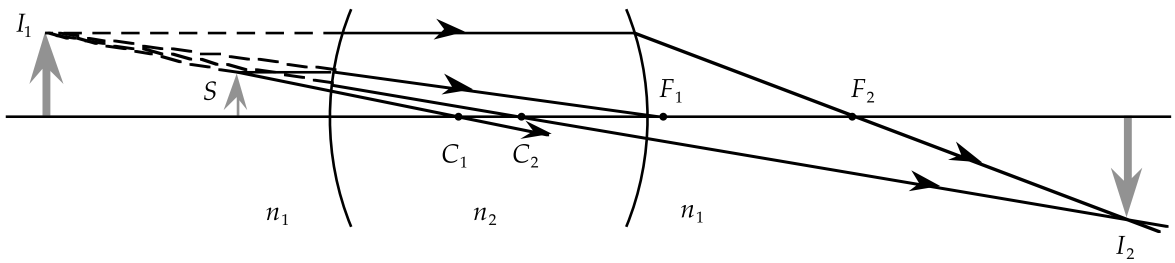
$1.6/(77 \text{ cm}) + 1.0/i_2 = (1.0 - 1.6)/(-18\text{cm})$, which gives $i_2 = -80$ cm.

The second image is 80 cm behind surface 2.

(c) The focal lengths of the two surfaces are

$f_1 = n_2R_1/(n_2 - n_1) = (1.6)(18 \text{ cm})/(1.60 - 1) = +48$ cm;

$f_2 = n_1R_2/(n_1 - n_2) = (1.0)(-18 \text{ cm})/(1.0 - 1.60) = +30$ cm.



The final image is inverted.

$\boxed{59}$. The focal lengths of the two mirrors are

$f_1 = R_1/2 = 16$ cm; $f_2 = R_2/2 = 7.0$ cm.

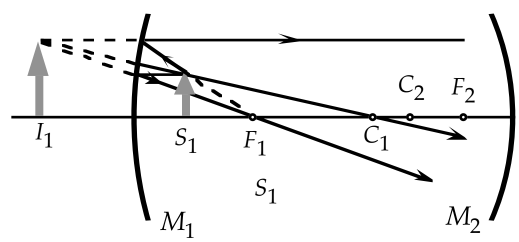
(a) For the first reflection, we have

$$\frac{1}{s_1} + \frac{1}{i_1} = \frac{1}{f_1};$$

$$\frac{1}{+7 \text{ cm}} + \frac{1}{i_1} = \frac{1}{+16 \text{ cm}}, \text{ which gives}$$

$i_1 = -12.4$ cm.

The first image is 12.4 cm behind M_1.



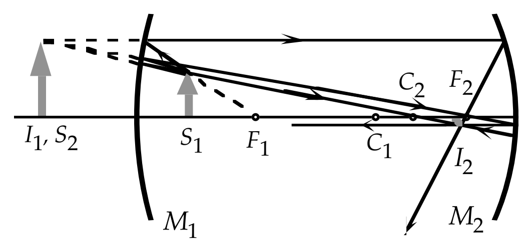
(b) The object distance for the second reflection is 62.4 cm, so we have

$$\frac{1}{s_2} + \frac{1}{i_2} = \frac{1}{f_2};$$

$$\frac{1}{+62.4 \text{ cm}} + \frac{1}{i_2} = \frac{1}{+7.0 \text{ cm}}, \text{ which gives}$$

$i_2 = +7.9$ cm.

The second image is 7.9 cm in front of M_2.



60. (a) For the first refraction, we have

$n_1/s_1 + n_2/i_1 = (n_2 - n_1)/R_1;$

$1.0/(62 \text{ cm}) + 1.5/i_1 = (1.5 - 1.0)/(20 \text{ cm}),$

which gives $i_1 = \boxed{+169 \text{ cm}}$.

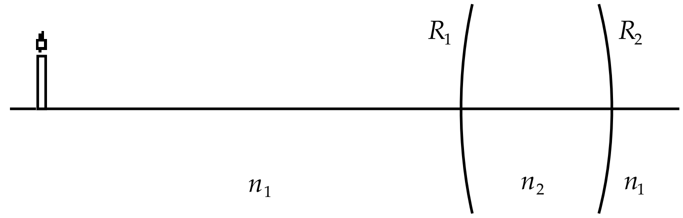
The first image is 169 cm behind the first surface.

The magnification is

$M_1 = -n_1 i_1/n_2 s_1$

$= -(1.0)(+169 \text{ cm})/(1.5)(+62\text{cm}) = \boxed{-1.8}$.

The image is inverted (as $M_1 < 0$).



(b) The first image is a virtual object 165 cm from the second surface. For the second refraction

$n_2/s_2 + n_1/i_2 = (n_1 - n_2)/R_2;$

$1.5/(-165 \text{ cm}) + 1.0/i_2 = (1.0 - 1.5)/(-10 \text{ cm})$, which gives $i_2 = \boxed{+17 \text{ cm}}$.

The second image is 17 cm behind the second surface, or 83 cm from the candle.

The magnification for the second refraction is

$M_2 = -n_2 i_2/n_1 s_2 = -(1.5)(+17 \text{ cm})/[(1.0)(-165 \text{ cm})] = \boxed{+0.15}$.

The overall magnification is

$M = M_1 M_2 = (-1.8)(+0.15) = \boxed{-0.28} < 0$, so the image is inverted.

61. We find the focal length by finding the image distance for an object very far away.

For the lens, we have

$1/s_1 + 1/i_1 = 1/f_1;$

$0 + 1/i_1 = 1/f_1$, or, as expected, $i_1 = f_1$.

The first image is the object for the mirror, with an object distance of $d - f_1$.

For the mirror, we have

$1/s_2 + 1/i_2 = 1/f_2;$

$1/(d - f_1) + 1/i_2 = 1/f_2$, which gives $i_2 = f_2(d - f_1)/[d - (f_1 + f_2)]$.

The reflected light must pass through the lens.

The second image is the object for the lens, with an object distance of $d - i_2$.

For the lens, we have

$1/s_3 + 1/i_3 = 1/f_1;$

$1/(d - i_2) + 1/i_3 = 1/f_1$.

Because this image must be at the focal length of the combination, we have

$1/f = 1/f_1 - 1/(d - i_2) = 1/f_1 - 1/[d - f_2(d - f_1)/(d - f_1 - f_2)]$.

After some algebra, we have

$f = \boxed{[f_1(d^2 - f_1 d - 2f_2 d + f_1 f_2)]/(d^2 - 2f_1 d - 2f_2 d + 2f_1 f_2 + f_1^2)}$.

62. For the refraction at the glass surface, we have

$(1.0) \sin\theta = n \sin\alpha = 2 \sin\alpha.$

For small angles, $\sin\phi \simeq \phi$, so this becomes

$\alpha = \frac{1}{2}\theta.$

If we add the angles of the triangle ACB, we have

$\alpha + \beta + (\pi - \theta) = \pi$, which gives

$\beta = \theta - \alpha = \theta - \frac{1}{2}\theta = \frac{1}{2}\theta.$

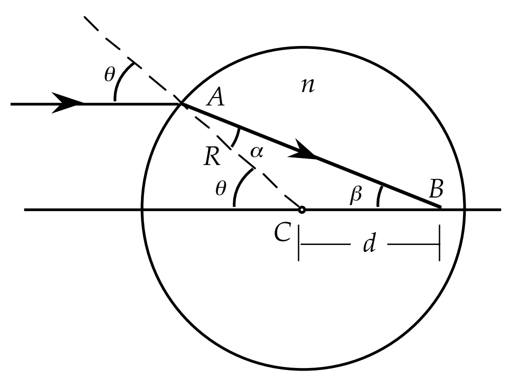
This shows that the triangle is isosceles, and

$CB = CA$, or $d = R$.

We could also use the refraction equation, since it was developed for paraxial rays:

$1/s + n/i = (n - 1)/R;$

$1/\infty + 2/i = (2 - 1)/R$, which gives $i = 2R$.



63. For the refraction at the glass surface, we have

$(1.0) \sin \theta = n \sin \alpha$.

If we add the angles of the triangle ACB, we have

$\alpha + \beta + (\pi - \theta) = \pi$, which gives $\beta = \theta - \alpha$.

We use the expansion for the sine of the sum of two angles:

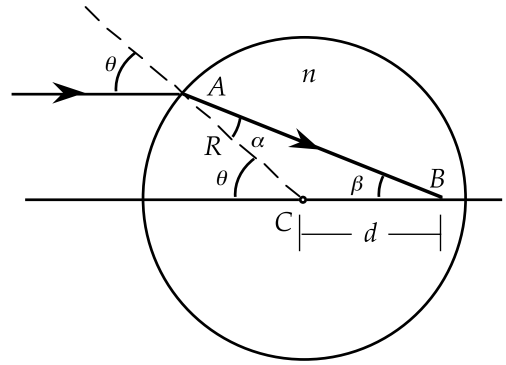
$\sin \beta = \sin(\theta - \alpha) = \sin \theta \cos \alpha - \cos \theta \sin \alpha$.

From the geometry of the triangle ACB, we have

$d/(\sin \alpha) = R/(\sin \beta)$.

For the distance, we have

$$\begin{aligned} d &= (R \sin \alpha)/(\sin \beta) \\ &= (R \sin \alpha)/(\sin \theta \cos \alpha - \cos \theta \sin \alpha) \\ &= (R \sin \alpha)/(n \sin \alpha \cos \alpha - \cos \theta \sin \alpha) \\ &= R/(n \cos \alpha - \cos \theta) \\ &= R/[n(1 - \sin^2 \alpha)^{1/2} - \cos \theta] \\ &= R/[(n^2 - n^2 \sin^2 \alpha)^{1/2} - \cos \theta] \\ &= R/[(n^2 - \sin^2 \theta)^{1/2} - \cos \theta]. \end{aligned}$$



For small angles, $\cos \theta \simeq 1$ and $\sin \theta \simeq 0$, so we have

$d \simeq R/[(n^2 - 0)^{1/2} - 1] = \boxed{R/(n-1)}$.

Note that this agrees with the result from Problem 56: When $n = 2$, $d = R$.

64. The angle of incidence is $\theta = 0.8$ rad $= 45.8°$. Using the result of Problem 63, we have

$d_{\text{blue}} = R/[(n_{\text{blue}}{}^2 - \sin^2 \theta)^{1/2} - \cos \theta]$

$= (1.50 \text{ cm})/[1.615^2 - \sin^2 45.8°)^{1/2} - \cos 45.8°] = \boxed{2.00 \text{ cm}}$;

$d_{\text{red}} = R/[(n_{\text{red}}{}^2 - \sin^2 \theta)^{1/2} - \cos \theta]$

$= (1.50 \text{ cm})/[(1.596^2 - \sin^2 45.8°)^{1/2} - \cos 45.8°] = \boxed{2.06 \text{ cm}}$.

The spread on the axis is $\boxed{\Delta d = 0.06 \text{ cm, which is} \simeq 3.0\% \text{ of average } d}$.

For paraxial rays, we have

$d_{\text{blue}} = R/(n_{\text{blue}} - 1) = (1.50 \text{ cm})/(1.615 - 1) = \boxed{2.44 \text{cm}}$;

$d_{\text{red}} = R/(n_{\text{red}} - 1) = (1.50 \text{ cm})/(1.596 - 1) = \boxed{2.52 \text{ cm}}$.

The spread on the axis is $\boxed{\Delta d = 0.08 \text{ cm, which is} \simeq 3.1\% \text{ of average } d}$.

65. We find the focal length of the lens from

$1/f = (n-1)(1/R_1 - 1/R_2) = (1.4 - 1)[1/(25 \text{ cm}) - 1/(-25 \text{ cm})]$, which gives $f = 31.25$ cm.

For a distant object, the image produced by the lens will be at the focal point: $i_1 = f = 31.25$ cm.

This image is a virtual object for refraction at the front surface of the plate. We let x be the distance from the lens to this surface. The object distance is $f - x$. For the radius of the flat surface, we use $R = \infty$:

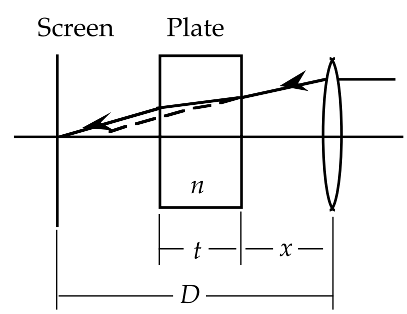
$1/s_2 + n/i_2 = (n-1)/\infty$;

$1/[-(f - x)] + n/i_2 = 0$, which gives $i_2 = n(f - x)$.

This image is a virtual object for refraction at the back surface of the plate. The object distance is $i_2 - t$. For the flat surface, we use $R = \infty$:

$n/s_3 + 1/i_3 = (1 - n)/\infty$;

$n/[-n(f - x) - t] + 1/i_3 = 0$, which gives $i_3 = f - x - t/n$.



For this final image to be on the screen, we have

$i_3 + t + x = D$;

$(f - x - t/n) + t + x = D$;

$f - t/n + t = D$;

$31.25 \text{ cm} - t/1.4 + t = 35 \text{ cm}$, which gives $t = 13.1$ cm.

Because the answer does not depend on x,

$\boxed{\text{the plate may be placed anywhere between the lens and the screen, with thickness 13.1 cm}}$.